

Dynamic critical behaviors in two-dimensional Josephson junction arrays with positional disorder

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We numerically investigate dynamic critical behaviors of two-dimensional (2D) Josephson-junction arrays with positional disorder in the scheme of the resistively shunted junction dynamics. Large-scale computation of the current voltage characteristics reveals an evidence supporting that a phase transition occurs at a nonzero critical temperature in the strong disorder regime, as well as in the weak disorder regime. The phase transition at weak disorder appears to belong to the Berezinskii-Kosterlitz-Thouless (BKT) type. In contrast, evidence for a non-BKT transition is found in the strong disorder regime. These results are consistent with the recent experiment on positionally disordered Josephson-junction arrays; in particular, the critical temperature of the non-BKT transition (ranging from 0.265 down to the minimum 0.22 in units of E_J/k_B with the Josephson coupling strength E_J), the correlation length critical exponent $\nu = 1.2$, and the dynamic critical exponent $z = 2.0$ in the strong disorder regime agree with the existing studies of the 2D gauge-glass model.

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I. INTRODUCTION

Phase transitions for the two-dimensional (2D) Josephson-junction arrays (JJAs) with weak positional disorder (i.e., on a slightly disordered lattice) and in a perpendicular magnetic field have attracted much attention. Here positional disorder in the presence of an external magnetic field effectively induces random phase shifts of the magnetic bond angles;^{1,2} thus the corresponding 2D random gauge XY (RGXY) model, in which the magnetic bond angles are quenched random variables distributed in a certain width, provides a theoretical realization of such a positionally disordered Josephson-junction array (PDJJA) in a magnetic field: Strong magnetic field in the PDJJA corresponds to strong disorder in the RGXY model. It has been observed that sufficiently weak disorder in the RGXY model does not destroy the quasi-long-range order present at low temperatures and accordingly, the system undergoes a Berezinskii-Kosterlitz-Thouless (BKT) type transition at a finite critical temperature T_c , which decreases as the disorder strength is raised up to a critical value.^{2,3,4} On the other hand, in spite of a number of studies, the strong disorder regime of the RGXY model (and its fully disordered limit corresponding to the gauge-glass model) has resisted adequate understanding. In parallel with the Mermin-Wagner theorem⁵ for the absence of the long-range order in the XY model, Nishimori⁶ has proven the absence of the long-range glass order in the gauge-glass model. However, it should be noted that the vanishing local glass order parameter in the gauge glass is not completely incom-

patible with the existence of the superconducting phase at nonzero temperatures. For instance, quasi-long-range glass order⁷ and possibility of a continuous phase transition of an anomalous dimension⁴ have been suggested.

On the one hand, there exist numerical evidences supporting the zero-temperature phase transition: Domain-wall renormalization-group studies^{2,3} predict that the RGXY model in the strong disorder regime as well as the gauge-glass model undergoes a phase transition only at zero temperature. For the gauge-glass model, the zero-temperature transition was supported by computations of various quantities such as the current-voltage (IV) characteristics,^{8,9} the root-mean-square current,^{10,11} the correlation length,^{12,13} the glass susceptibility,^{10,12,13} the autocorrelation function,^{11,13} and the phase slip resistance.¹¹ A recent numerical consideration of the low-energy excitations also estimated $T_c = 0$ in the gauge-glass model.¹⁴

In contrast, the finite-size scaling analysis applied to the helicity modulus and the root-mean-square current in the RGXY model with strong disorder demonstrated that the superconducting phase transition of a non-BKT type occurs at finite T_c , independent of the disorder strength.⁴ In fact evidences for such a finite-temperature transition in the gauge-glass model have been presented in a few numerical studies computing the IV characteristics,^{15,16} the correlation function,⁷ the glass susceptibility,⁷ and the linear resistance.¹⁷ Here it should be noted that critical exponents and T_c obtained from the resistively shunted junction (RSJ) simulations in large scales¹⁶ agree with other numerical studies of the gauge

glass,^{7,17} and lead to $T_c = 0.22$ (in units of E_J/k_B , where E_J is the coupling energy and k_B the Boltzmann constant), the correlation length critical exponent $\nu = 1.2$, and the dynamic critical exponent $z = 2.0$. Furthermore, the barrier energy and the associated vortex mobility in the gauge glass¹⁸ implies that superconducting order persists at low but finite temperatures.

Very recently, an experiment has been performed on the PDJJA,¹⁹ and the critical temperature has been measured as a function of the disorder strength, which reveals a finite-temperature transition at strong disorder. Further, in the experiment¹⁹ the scaling behavior of the IV characteristics is observed consistent with the corresponding numerical results for the gauge glass,^{15,16} indicating the presence of a non-BKT type transition at a finite temperature in the strong disorder regime. A truly remarkable feature of this PDJJA experiment is that a very clear signal of a finite-temperature transition has been obtained for a *single* disorder realization of a 200×800 square array. In contrast, typical attempts to settle the issue of the existence of a finite-temperature transition by direct simulations usually presume that a well converged disorder average is necessary. This in turn limits the simulations to small system sizes, so that any conclusion crucially depends on *a priori* assumptions for the finite-size scaling. The PDJJA experiment in Ref. 19 suggests that one could avoid such *a priori* assumptions by instead obtaining well converged data for large samples at the price of focussing on a single disorder realization.

In the present paper we test this alternative strategy by performing RSJ simulations and computing the IV -characteristics of a large square array of Josephson junctions with one randomly chosen positional disorder realization. In parallel with the experimental results in Ref. 19, we find strong numerical evidence for a finite-temperature transition. In view of the on-going controversy as to the existence of such a phase transition, we discuss the possible implications of our results.

The present paper is organized as follows: In Sec. II, we briefly describe the numerical method used in this work, and discuss the relation between the PDJJA and the RGXY model. Section III presents mostly the results for the PDJJA at weak disorder, which is followed by the results at strong disorder in Sec. IV. In Sec. V, we summarize the results of the present paper in relation with the existing studies.

II. MODEL AND SIMULATIONS

With the assumption that the coupling energy is not affected by positional disorder, the Hamiltonian of the PDJJA in the presence of a transverse magnetic field reads²⁵

$$H = -E_J \sum_{\langle ij \rangle} \cos(\phi_{ij} - A_{ij}), \quad (1)$$

where ϕ_{ij} is the phase difference between the superconducting islands at sites i and j . When an external magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ is applied, the magnetic bond angle A_{ij} obtains the form

$$A_{ij} = \frac{Ba^2\pi}{\Phi_0} (x_j + x_i)(y_j - y_i), \quad (2)$$

where a is the lattice constant and Φ_0 is the magnetic flux quantum. Due to the positional disorder, the position of the i th island is given by

$$\mathbf{r}_i \equiv (x_i, y_i) = (x_i^0 + \delta x_i, y_i^0 + \delta y_i), \quad (3)$$

where $\mathbf{r}_i^0 \equiv (x_i^0, y_i^0)$ represents the ideal position without disorder and δx_i and δy_i are random quenched variables uniformly distributed in the interval $[-\Delta, \Delta]$. Henceforth, \mathbf{r}_i , x_i , y_i , δx_i , and δy_i are all taken to be dimensionless, measured in units of the lattice spacing a .

The magnetic frustration f is usually defined as the number of flux quanta per plaquette. There has been intensive research interest in the role of the magnetic frustration in both classical²⁰ and quantum²¹ systems. In the absence of positional disorder, every plaquette has an equal area, thus f is constant over the whole system. It is well known that the Hamiltonian (1) in this case is invariant under the transformation $f \rightarrow f \pm 1$.²⁰ In the present work, however, the plaquette area changes from place to place, and the Hamiltonian loses the above symmetry, invalidating the equivalence between the cases f and $f \pm 1$. We in this work define f as the *disorder average* (denoted to be $[\dots]$) of the flux through one plaquette

$$\left[\sum_p A_{ij} \right] = 2\pi \frac{Ba^2}{\Phi_0} \equiv 2\pi f. \quad (4)$$

The magnetic bond angle A_{ij} in Eq. (2) is correlated with the nearest neighboring bond angles since a change of position \mathbf{r}_i gives rise to changes of four magnetic bond angles A_{ij} with site j being neighbors of site i . Without such short-range correlations, A_{ij} becomes a random quenched variable characterized by the disorder average $[A_{ij}] = 2\pi f (x_j^0 + x_i^0)(y_j^0 - y_i^0)$. In case that f is an integer, one can gauge away the magnetic bond angle, establishing the equivalence with the RGXY model where the average frustration across the whole system vanishes. The variance of the sum of the magnetic bond angles around one plaquette in a PDJJA is given by

$$\left[\left(\sum_p A_{ij} \right)^2 \right] - \left[\sum_p A_{ij} \right]^2 = \pi^2 f^2 \left(\frac{4}{3} \Delta^2 + \frac{8}{9} \Delta^4 \right) \quad (5)$$

whereas the corresponding quantity in the RGXY model reads

$$\left[\left(\sum_p A_{ij} \right)^2 \right] - \left[\sum_p A_{ij} \right]^2 = \frac{4\pi^2}{3} r^2, \quad (6)$$

where A_{ij} has been taken to be uniformly distributed in $[-r\pi, r\pi]$ with the disorder strength r . It is to be noted that unless Δ is close to unity, one obtains

$$r \approx f\Delta. \quad (7)$$

Consequently, the disorder strength of the PDJJA in comparison with the RGXY model is measured by $f\Delta$, as found in previous studies of the PDJJA.^{22,23} It should be noted that the equivalence between the PDJJA and the RGXY model becomes valid only when the external magnetic field yields an integer value of the average magnetic frustration. The relation in Eq. (7) is practically very useful since it provides a convenient way to perform experiments on the 2D RGXY model, in which the disorder strength can be tuned for one sample only by increasing the integral value of f .

We below sketch briefly the numerical method adopted in this work. Introducing the twist variable $\mathbf{D} \equiv (D_x, D_y)$ for the fluctuating twist boundary conditions (FTBC)²⁴ in $L \times L$ arrays, we write ϕ_{ij} in Eq. (1) as

$$\phi_{ij} = \theta_i - \theta_j - \mathbf{r}_{ij}^0 \cdot \mathbf{D}, \quad (8)$$

where $\mathbf{r}_{ij}^0 \equiv \mathbf{r}_j^0 - \mathbf{r}_i^0$ and θ_i is the phase angle. Since the effects of positional disorder are introduced only through magnetic bond angles, we use the RSJ dynamics combined with the FTBC as follows: Using the local current conservation, the equation for the phase angle at site i is given by

$$\dot{\theta}_i = - \sum_j G_{ij} \sum_k' [\sin(\phi_{jk} - A_{jk}) + \eta_{jk}], \quad (9)$$

where time has been measured in units of $\hbar/2e i_c R$ with the single-junction critical current $i_c = 2eE_J/\hbar$ and the shunt resistance R (see Ref. 24 for details), G_{ij} is the square-lattice Green function, \sum_k' denotes the summation over the four nearest neighbor sites of j , and η_{jk} is the dimensionless thermal noise current satisfying $\langle \eta_{ij} \rangle = 0$ and

$$\langle \eta_{ij}(t) \eta_{kl}(0) \rangle = 2T (\delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk}) \delta(t), \quad (10)$$

with the temperature T in units of E_J/k_B . In order to use the efficient fast Fourier transform, we modify the periodic boundary condition for θ_i according to

$$\begin{aligned} \theta_{i+L\hat{x}} &= \theta_i + 2\pi f L \delta y_i \\ \theta_{i+L\hat{y}} &= \theta_i, \end{aligned} \quad (11)$$

where $2\pi f L \delta y_i$ is fixed in time and thus $\dot{\theta}_{i+N\hat{x}} = \dot{\theta}_i$. Here the usual periodic boundary conditions ($\theta_{i+L\hat{x}} = \theta_{i+L\hat{y}} = \theta_i$) are not applicable since $A_{ij} \neq A_{i+L\hat{x}, j+L\hat{x}}$ for the PDJJA.

We consider the system under uniform external currents injected along the x -direction. The Josephson relation $2eV_x/\hbar = \dot{\phi}_{i+N\hat{x}, i}$, where V_x is the voltage drop

across the sample, together with the global current conservation condition,²⁴ leads to the equations of motion for the twist variables D_x and D_y in the form

$$\begin{aligned} \dot{D}_x &= \frac{1}{L^2} \sum_{\langle ij \rangle_x} \sin(\phi_{ij} - A_{ij}) + \eta_{D_x} - i_d, \\ \dot{D}_y &= \frac{1}{L^2} \sum_{\langle ij \rangle_y} \sin(\phi_{ij} - A_{ij}) + \eta_{D_y}, \end{aligned} \quad (12)$$

where $\sum_{\langle ij \rangle_{x(y)}}$ denotes the directed sum over nearest neighboring bonds in the $x(y)$ direction, i_d is the external (driving) current density in units of the single-junction critical current i_c , and η_{D_x} (η_{D_y}) is the thermal noise current for D_x (D_y) satisfying $\langle \eta_{D_x} \rangle = \langle \eta_{D_y} \rangle = 0$ and

$$\langle \eta_{D_x}(t) \eta_{D_x}(0) \rangle = \langle \eta_{D_y}(t) \eta_{D_y}(0) \rangle = 2(T/L^2) \delta(t). \quad (13)$$

We numerically integrate the equations of motion given by Eqs. (9) and (12), and compute the IV characteristics with the average (dc) voltage $V \equiv Lv \equiv -L \langle \dot{D}_x \rangle$, where v denotes the voltage drop per junction in units of $i_c R$.

III. RESULTS FOR WEAK DISORDER

Our simulation scheme requires sufficiently precise data for a large sample and a large disorder strength. We find that RSJ simulations of the PDJJA on a 128×128 square lattice can be made to meet all these three requirements simultaneously, and thus use this lattice size in the present investigation.

To probe phase transitions in weak and strong disorder regimes, we compute the IV characteristics at various temperatures and values of f with the latter controlling the disorder strength (see Sec. II). In our simulations we set the parameter for positional disorder equal to $\Delta = 0.2$. Since $r \approx f\Delta$ and the critical disorder strength in the RGXY model is believed to be $r_c \approx 0.4$,^{2,3,4} we vary the average frustration f from 1 to 4 to cover both weakly ($f = 1$ with $f\Delta \approx 0.2 < r_c$) and strongly ($f \geq 2$ with $f\Delta > r_c$) disordered cases.

Figure 1 presents the dc resistance v/i_d (in units of the shunt resistance R) versus the temperature T at various frustrations $f = 1, 2, 3$, and 4. When $f = 1$, corresponding to the weakly disordered case, the resistance-temperature (RT) curves in Fig. 1 disclose that the system undergoes a phase transition around $T = 0.6$, which is lower than $T_c = 0.89$ in a regular JJA.²⁴ For strong disorder ($f \geq 2$), phase transitions are observed to occur near $T = 0.2$, hardly depending on the frustration.

In order to determine the transition temperature in a more accurate way and to understand the dynamic critical behavior in detail, we next investigate the IV characteristics at various values of f with the help of the scaling form suggested in Ref. 26. In two dimensions, the current density and the electric field behave as $J \sim T/\xi$ and $E \sim \xi^{-1-z}$, respectively, with the correlation length ξ .

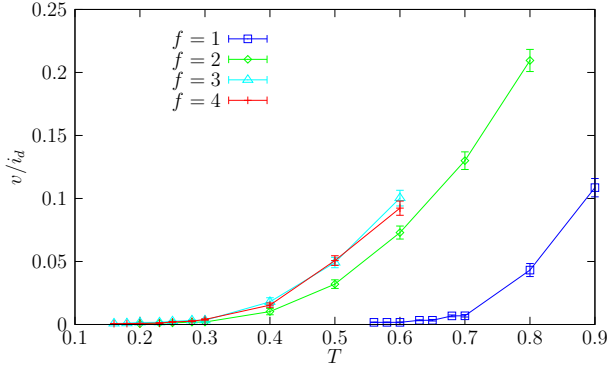


FIG. 1: (Color online) Resistance v/i_d (in units of R) versus temperature T (in units of E_J/k_B) for various frustrations in positionally disordered Josephson-junction arrays. As the frustration is raised from $f = 1$ to $f = 2$, the effective disorder strength becomes larger, reducing the critical temperature. Larger values of f ($= 3, 4$) yield only insignificant variations of the critical temperature.

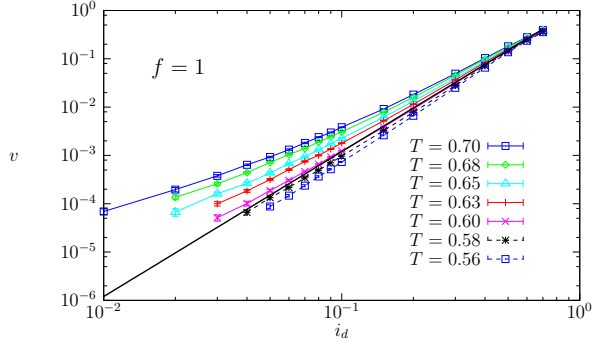


FIG. 2: (Color online) IV curves (v versus i_d) for $f = 1$ in log scales at various temperatures T (in units of E_J/k_B). At $T \lesssim 0.6$, curves fit well to the power-law form $v \approx i_d^{z+1}$, with the solid line representing the case $z = 2$. At higher temperatures, IV curves bend upwards, indicating a BKT-type transition.

In the case $T_c \neq 0$, one replaces T in $J \sim T/\xi$ by T_c near criticality, and obtains accordingly $J \sim \xi^{-1}$. In this work, i_d and v correspond to the current density and the electric field, respectively, and yield the scaling form

$$v = i_d \xi^{-z} F_{\pm}(i_d \xi) \quad (14)$$

with the scaling function F_{\pm} above/below T_c . At $T = T_c$, the correlation length ξ diverges and Eq. (14) leads to $v \approx i_d^{z+1}$.

In Fig. 2, we display the IV characteristics for $f = 1$, corresponding to the weak disorder regime, near $T = T_c \approx 0.6$ (see the RT curve for $f = 1$ in Fig. 1). Due to the finite-size effects, IV curves exhibit the Ohmic behavior in the small current region as $i_d \rightarrow 0$.²⁴ The curves at $T \lesssim 0.6$ fit well to the power-law form $v \sim i_d^a$: At $T \approx 0.6$, we have $a \approx 3$, and the lower T , the larger a . In contrast, the IV curves at higher temperatures ($T \gtrsim 0.6$) clearly exhibit upward curvature, indicating the existence

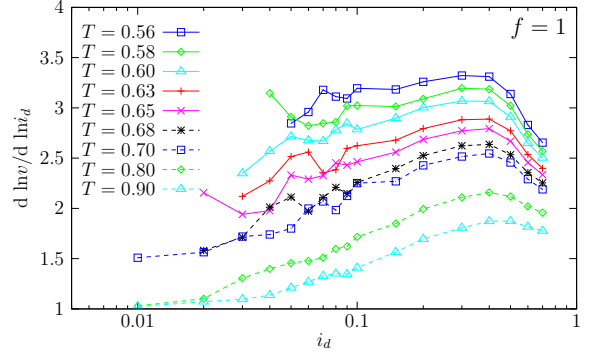


FIG. 3: (Color online) Slope $d \ln v / d \ln i_d$ of the IV curve versus the driving current i_d at various temperatures (in units of E_J/k_B). Due to finite-size effects, the slope first increases with i_d , then decreases toward the Ohmic value, unity, thus forming a peak. In a broad range of temperatures below $T_c \approx 0.7$, the peak position i_d^* does not change substantially while it shifts to higher currents beyond T_c . Such behavior of the peak position is consistent with the BKT-type transition.

of a finite current scale (and thus of a finite length scale). From the relation $a = z + 1$ between the dynamic critical exponent z and the nonlinear IV exponent a , we thus reach the conclusion that a phase transition of the BKT nature occurs at $T_c \approx 0.6$ with $z \approx 2$. It should be noted that the scaling form in Eq. (14) is valid when the correlation length diverges only at T_c and becomes smaller as the temperature is varied (either lowered or raised) from T_c . Accordingly, when the phase transition is of the BKT type, characterized by the diverging length scale in the whole low-temperature phase, only high-temperature IV curves collapse to the scaling form, while the low-temperature part cannot be made to collapse due to the lack of the length scale ξ .

Alternatively, one can also plot the slopes of IV curves, given by $d \ln v / d \ln i_d$, versus the driving current i_d , as shown in Fig. 3 to confirm the BKT nature of the transition.²⁷ As i_d is reduced, all curves should eventually crossover to the Ohmic behavior characterized by the unit slope ($d \ln v / d \ln i_d = 1$) due to the finite-size effects. The peak position i_d^* of each curve in Fig. 3 measures a characteristic current scale, which is inversely proportional to the length scale in the system, i.e., $i_d^* \approx 1/\xi$. The correlation length ξ increases as T approaches T_c from high temperatures. Near T_c in the high-temperature phase and also in the whole low-temperature phase, the correlation length becomes larger than the size of the system. In this case, the relevant length scale of the system is not the correlation length but the linear size L of the system, leading to $i_d^* \sim 1/L$ independent of the temperature. In Fig. 3, the peak position i_d^* has almost the same value around 0.4 as T is increased from below up to $T \approx 0.7$; then i_d^* appears to drift away toward larger values as T is increased further beyond $T = 0.7$, which is in good agreement with what we expect for the BKT-type phase transition. We emphasize again that for a finite

system i_d^* starts to move not at the true critical temperature but at a temperature higher than T_c when the correlation length becomes comparable to L . Between $T = 0.63$ and 0.6 , $d \ln v / d \ln i_d \approx 3$ at i_d^* , which, combined with the power-law decay form in Fig. 2, makes us conclude that the BKT transition occurs at $T_c \approx 0.6$ (in accord with the RT curve in Fig. 1) with the dynamic critical exponent $z \approx 2$. This analysis of the IV curves clearly reveals that the phase transition of the BKT type occurs in the weak positional disorder regime of the PDJJA, consistent with experiment¹⁹ and previous numerical studies of RGXY model.^{2,3,4}

IV. RESULTS FOR STRONG DISORDER

In the strong disorder regime ($f \geq 2$), we show in Fig. 4 the IV curves for $f =$ (a) 2, (b) 3, and (c) 4. It is clearly observed that for each value of f , there exists a well-defined temperature T_c at which the IV curve shows a power-law behavior, manifesting the absence of a length scale at criticality. The critical temperature estimated from Fig. 4 is $T_c =$ (a) 0.25, (b) 0.23, and (c) 0.23 for $f = 2, 3$, and 4, respectively, in agreement with Fig. 1. It is also shown in Fig. 4 that the IV curves at T_c fit well to the form i_d^{z+1} with the dynamic critical exponent $z = 2$. The IV curves in Figs. 4 (a), (b), and (c) all bend upward above T_c , indicating that the PDJJA is in the high-temperature normal phase, while the opposite downward curvature below T_c implies the superconducting phase in the limit of $i_d \rightarrow 0$.

To find nature of the phase transition and the critical temperature together with critical exponents in the strong disorder regime of the PDJJA, we employ the scaling form in Eq. (14), which, with the correlation length $\xi \sim |T - T_c|^{-\nu}$, reads

$$\frac{v}{i_d |T - T_c|^{z\nu}} = F_{\pm}(i_d |T - T_c|^{-\nu}). \quad (15)$$

Figures 4 (d)-(f) show that data points in the IV characteristics [in Figs. 4 (a)-(c)] collapse into two function F_{\pm} in Eq. (15); this confirms that for given value of f , there exists a finite-temperature phase transition of a non-BKT type. The critical temperature changes from $T_c \approx 0.265$ for $f = 2$ to $T_c \approx 0.22$ for $f = 3$ and 4, which is consistent with the experiment on the PDJJA¹⁹ and numerical studies of the RGXY model⁴ and the gauge-glass model.^{7,15,16,17,18} On the other hand, in all three cases [$f = 2, 3$, and 4 in Figs. 4 (d)-(f)], the scaling collapse is achieved with $\nu = 1.2$ and $z = 2.0$, implying that the nature of the transition in the strong disorder regime remains unchanged as the disorder strength is increased. Furthermore, the dynamic critical exponent $z = 2.0$ agrees with the studies of dynamic behavior in the gauge-glass model^{7,16,17} as well as with the experimental results¹⁹. The critical exponent $\nu = 1.2$ obtained in this work is again consistent with the previous numerical results^{4,7,16,17} but not with the experimental result

for the PDJJA,¹⁹ the origin of which is not clear at this stage.

We next investigate slopes of the IV curves for $f = 4$ in the same manner as in Sec. III for the weak disorder case for $f = 1$. In Fig. 5, the slope $d \ln v / d \ln i_d$ is plotted as a function of i_d at various temperatures around T_c . Note that finite-size effects yield the Ohmic behavior, $d \ln v / d \ln i_d \approx 1$, as $i_d \rightarrow 0$. It is observed that the curves at $T = 0.23$ and 0.20 are almost horizontal in broad ranges of the external currents, indicating scale-free behavior and in turn the existence of a phase transition with the dynamic critical exponent $z \approx 2.0$, consistent with the finding in Fig. 4(f) for $f = 4$. Furthermore, the fact that the peak position changes rather abruptly from large currents to smaller currents at $T = 0.23$ to 0.20 implies that the phase transition is of a non-BKT type, namely, the IV data are well scaled to two different functions, F_{\pm} in Eq. (15) above/below T_c .

In view of the ongoing controversy about the existence of a finite-temperature transition of a non-BKT character for strong disorder, one might ask what our results really imply. First of all, our evidence has been obtained for a finite sample with one disorder realization (albeit a very large one in comparison with those in most of earlier investigations). Does the phase transition survive in the large system size limit? Since a single positional disorder for a finite system can be periodically repeated, we can obtain an arbitrarily large system by just adding new squares with the same single disorder realization. Such an infinite system constructed from a single disorder realization will, to our belief, most certainly have a finite-temperature transition of a non-BKT type. This means that the phase transition does exist *per se*. Suppose that we instead choose an arbitrarily large system and generate the disorder in the same random way as we have done for our 128×128 sample. Would the phase transition still survive? Here we find that various randomly generated disorder realizations for the 128×128 system yield numerically very similar results, indicating that for the size $L = 128$ there is already a large amount of disorder self-averaging. This again suggests that our results will survive in the large L limit, i.e., for the PDJJA with uniformly distributed disorder, chosen randomly, of strength $\Delta = 0.2$. In order to check the finite-size effect on the self-averaging property in a more careful way, we have also computed voltages at $T = 0.16$ in the strong disorder regime ($f = 4$) for smaller sizes ($L = 16, 32, 64$, and 128) at two different disorder realizations: The difference between voltages obtained from different disorder realizations is found to decrease as L is increased, and becomes negligibly small at $L = 128$. This indicates that indeed the self-averaging effect becomes clear beyond $L = 128$. The next question is then whether our results also carry over to the RGXY model and the gauge-glass model. Here the evidence is more circumstantial and based, on the one hand, on the strong connection between the PDJJA and the RGXY and random gauge-glass models^{1,2} and, on the other hand, on the

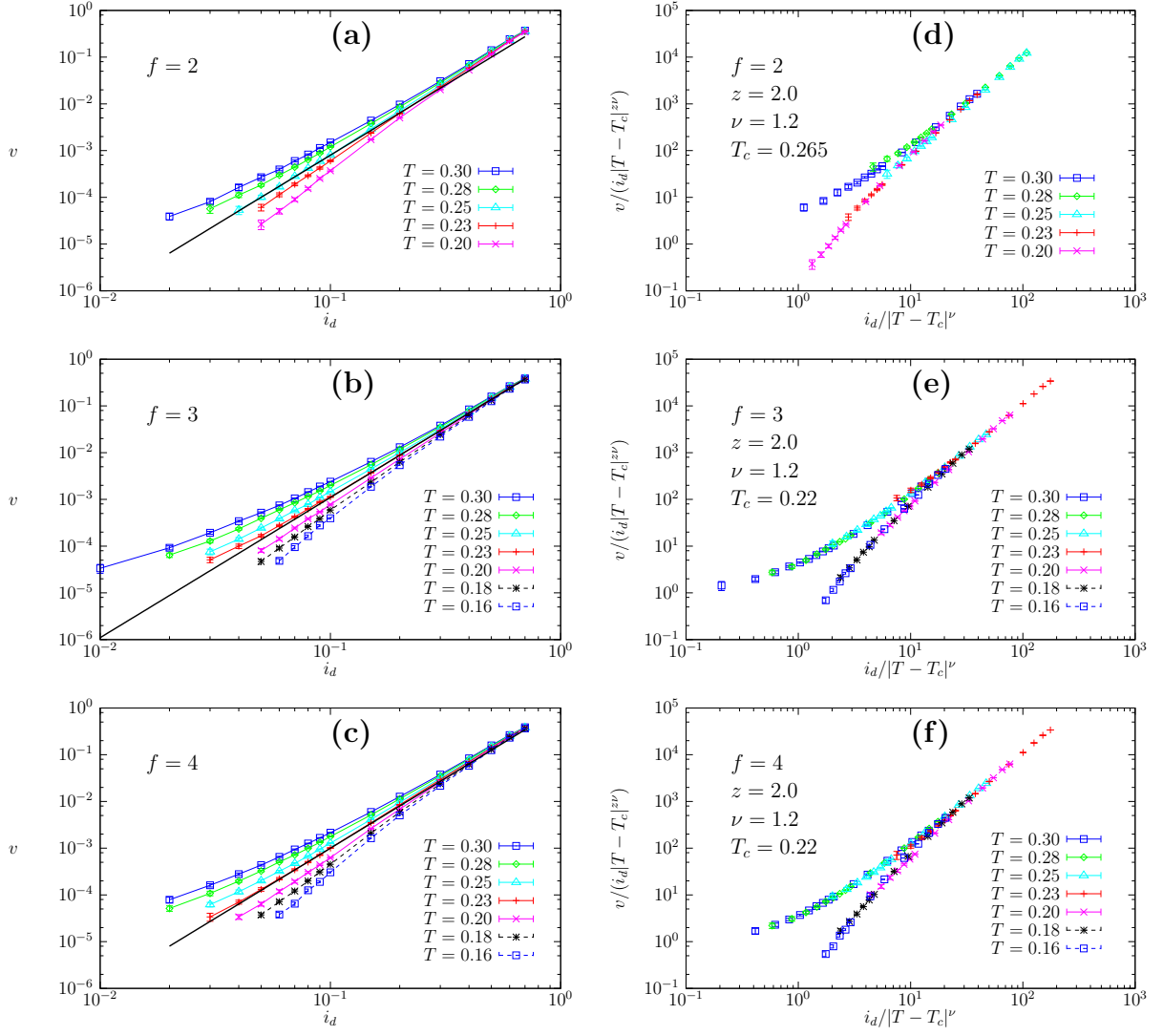


FIG. 4: (Color online) IV characteristics (v versus i_d) at various temperatures for $f =$ (a) 2, (b) 3, and (c) 4, and the corresponding scaling plots in (d)-(f). The solid lines in (a), (b), and (c) represent the power-law decay form $v \approx i_d^{z+1}$ with $z = 2$. A well-defined temperature T_c separates the IV curves into two groups, one bending upwards and the other downwards. All data points in the corresponding IV characteristics are made to collapse into scaling functions, as shown in (d)-(f), with $T_c = 0.265, 0.22$, and 0.22 , respectively. In all the three cases (d)-(f), the same critical exponents $\nu = 1.2$ and $z = 2.0$ are used, implying that the transitions in the strong disorder regime ($f = 2, 3$, and 4) belong to the same universality class.

strong similarity of the present results to some earlier results obtained from the latter models.^{4,7,16,17}

V. SUMMARY

The existence of a finite-temperature phase transition in the strong disorder regime of the PDJJA and the 2D RGXY model, including the fully disordered case of the 2D gauge-glass model, are still in an intensive debate. This work has been motivated by the very recent experiment on 800×200 Josephson junction arrays with positional disorder,¹⁹ and explored numerically the dynamic critical behavior of the PDJJA in transverse

magnetic fields. Adopting the RSJ dynamics,²⁴ we have computed the IV characteristics of the PDJJA with the positional disorder parameter $\Delta = 0.2$ for frustration $f = 1, 2, 3$ and 4. The relation $r \approx f\Delta$ between the disorder strength r in the 2D RGXY model and the parameter Δ in the PDJJA, combined with the critical disorder strength $r_c \approx 0.4$,^{2,3,4} implies that the PDJJA for $f = 1$ corresponds to the weakly disordered case while strong frustration ($f \gtrsim 2$) puts the PDJJA in the strong disorder regime. The scaling analysis²⁶ of IV curves and their slopes²⁷ has revealed clear evidence for $T_c \neq 0$ in the PDJJA with strong disorder, which agrees with the experiment on the PDJJA¹⁹ as well as previous numerical studies of the RGXY model⁴ and the gauge-glass

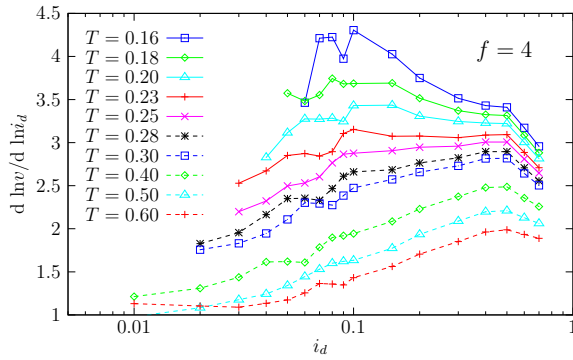


FIG. 5: (Color online) Slope ($d \ln v / d \ln i_d$) of IV curves against i_d for $f = 4$, corresponding the strong disorder regime of the PDJJA, at various temperatures T (in units of E_J/k_B). Power-law behavior appears at $T = 0.23$ to 0.20 with the dynamic critical exponent $z \approx 2$, indicating the existence of a phase transition with a diverging length scale only at T_c .

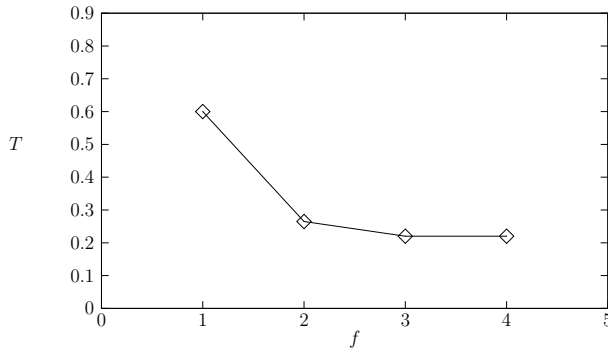


FIG. 6: Phase diagram on the plane of temperature T (in units of E_J/k_B) and frustration f (controlling the disorder strength). The solid line, separating the high-temperature normal phase from the low-temperature superconducting phase, is only a guide to the eye. The critical temperature saturates to a finite nonzero value $T_c \approx 0.22$ as f is increased. Below T_c , depending on the disorder strength, there exist two different superconducting phase: a BKT-like phase for $f = 1$ and a non-BKT type one for $f \gtrsim 2$.

model.^{7,15,16,17,18}

Figure 6 exhibiting the phase diagram on the plane of the temperature and frustration summarizes the results of the present work. The critical temperature T_c is observed to reduce rapidly as the average frustration f , controlling the disorder strength, is increased in the weak disorder region. It then appears to saturate toward a constant value $T_c = 0.22$ in the strong disorder regime. Below the phase boundary separating the superconducting phase at low temperatures and the normal phase at high temperature, there exist two different superconducting orders, according to the disorder strength: In the weak disorder regime, e.g., $f = 1$, the superconducting state is the low-temperature BKT phase characterized by the divergence of the correlation length. On the other hand, for strong disorder ($f \gtrsim 2$), the transition is of the non-BKT type with the well-defined correlation length critical exponent $\nu = 1.2$, consistent with the value obtained previously for the 2D gauge-glass model^{7,16,17} but inconsistent with the experimental finding $\nu = 2.0 \pm 0.3$.¹⁹ The origin of this discrepancy is not clear at present and needs more detailed investigation in the future study. Finally, we point out that the resemblance between the phase diagram in Fig. 6 and the corresponding diagram obtained in the recent experiment on the PDJJA¹⁹ is striking, which is also in very good agreement with the numerical study of the 2D random gauge XY model.⁴

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